GCE AS MARKING SCHEME

SUMMER 2018

AS (NEW)
FURTHER MATHEMATICS
UNIT 2 FURTHER STATISTICS A
2305U20-1

INTRODUCTION

This marking scheme was used by WJEC for the 2018 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE Further Mathematics – AS Unit 2 Further Statistics A

Solutions and Mark Scheme

SUMMER 2018 MARK SCHEME

Qu. No.	Solution	Mark	Notes
1(a)	$E(X) = 3.6$ and $E(Y) = 4$	B1	Both seen or implied in (a) or (b)
	$E(XY)(= 3.6 \times 4)$		
	$= 14.4$	B1	
(b)	$Var(X) = 2.52$ and $Var(Y) = 4$	B1	Both si
	$E(X^2) = Var(X) + (E(X))^2$		
	$= 2.52 + 3.6^2$	M1	Correct method for
	$= 15.48$	A1	either $E(X^2)$ or $E(Y^2)$
	$E(Y^2) = 4 + 4^2$		
	$= 20$	A1	
	$Var(XY) = E(X^2)E(Y^2) - (E(XY))^2$		
	$= 15.48 \times 20 - 14.4^2$	m1	Dep on previous M1 FT their 14.4, 15.48 and 20 for m1 only.
	$= 102.24$	A1	cao
		[8]	

Qu. No.	Solution	Mark	Notes
2(a)	$P(X > 5) = 1 - F(5)$ $= 0.1319(44\ldots)$ or $\frac{19}{144}$ **awrt** 0.132	M1 A1	
(b)	P(torch will operate for more than 50 hours) $= 0.13194444\ldots^3$ $= 0.00229(70....)$ **awrt** 0.0023	M1 A1	'Their (a)'^{3}
(c)	$F(4.5) = 0.7382$ ... $F(4.6) = 0.7660$ ... Since $F(4.6)$ is greater than 0.75 and $F(4.5)$ is less than 0.75 the solution to $F(q) = 0.75$ is between 4.5 and 4.6	M1 A1 E1	M1 for attempt to find $F(4.5)$ and $F(4.6)$ A1 for both answers. If rearranged to $q^4 - 8q^3 + 324 = 0$ A1 is for 5.0625 and -6.9424. Accept oe
(d)	$f(x) = F'(x)$ $f(x) = \frac{8 \times 3x^2}{432} - \frac{4x^3}{432}$ $f(x) = \begin{cases} \frac{x^2}{108}(6-x) & 0 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$ $f(0)$ e. $x =$	M1 A1 B1	M1 Attempt at differentiating with at least one power of x decreasing A1 Correct expression for $f(x)$ for x between 0 and 6. B1 for "0 otherwise" and range $0 \le x \le 6$
(e)	$E(X) = \int_0^6 \frac{x^3}{108}(6-x)dx$ $E(X) = \frac{1}{108}\int_0^6 (6x^3 - x^4)dx$ $E(X) = \frac{1}{108}\left[\frac{6x^4}{4} - \frac{x^5}{5}\right]_0^6$ $E(X) = 3.6$ Mean = 36 hours	M1 A1 A1 B1	M1 Attempt at integrating $xf(x)$ with at least one power of x increasing FT 'their $f(x)$' of equivalent difficulty (ignore limits here) A1 correct integration with correct limits FT cao FT their derived $E(X)$
(f)	Valid explanation e.g. It is possible for a battery to last more than 60 hours. e.g. X could be greater than 6.	E1 **[15]**	

Qu. No.	Solution	Mark	Notes
3 (a)	Let the random variable X be the player's profit in pence. Values for x are -50, 50 and 450 $P(X=-50)=\frac{1}{2}+\frac{1}{2}\times\frac{39}{52}\left(=\frac{91}{104}=\frac{7}{8}=0.875\right)$ OR $P(X=50)=\frac{1}{2}\times\frac{12}{52}\left(=\frac{12}{104}=\frac{3}{26}=0.115\ldots\right)$ OR $P(X=450)=\frac{1}{2}\times\frac{1}{52}\left(=\frac{1}{104}=0.00961\ldots\right)$	B1 M1 A1 for $\frac{7}{8}$ oe A1 for $\frac{3}{26}$ oe A1 for $\frac{1}{104}$ oe	B1 for all three values. M1 for correct working for $P(X=50)$ or $P(X=450)$ or $P(X=-50)$ Accept answers in £ or pence for this question. Only award final A1 if all correct probabilities are associated with the correct,corresponding values of x.
	x | -50 | 50 | 450 $P(X=x)$ | $\frac{91}{104}$ | $\frac{12}{104}$ | $\frac{1}{104}$	A1 A1 A1	
	OR x | -50 | 50 | 450 $P(X=x)$ | $\frac{7}{8}$ | $\frac{3}{26}$ | $\frac{1}{104}$		
(b)	$E(X)=-50\times\frac{7}{8}+50\times\frac{3}{26}+450\times\frac{1}{104}=\frac{-875}{26}$ = -33.65 (pence) OR $\frac{-875}{26}$ **awrt -33.7** $E(X^2)=(-50)^2\times\frac{7}{8}+50^2\times\frac{3}{26}+450^2\times\frac{1}{104}$ $Var(X)=(-50)^2\times\frac{7}{8}+50^2\times\frac{3}{26}+450^2\times\frac{1}{104}-\left(\frac{-875}{26}\right)^2$ $\sigma=\sqrt{3290.495562}$ = 57.36(284… pence)	M1 A1 M1 A1	FT their probability distribution for M1A1. $\frac{-35}{104}$ if working in £
(c)(i)	$\left(\frac{1}{8}\times 200=\right)$ 25 players	B1	
(ii)	$\left(\frac{875}{26}\times 200=\right)$ £67.31	B1 **[11]**	Accept £67.30 FT their –E(X)

Qu. No.	Solution	Mark	Notes
4 (a)	The ranks are | Cow | A | B | C | D | E | F | G | | Actual weight | 7 | 1 | 5 | 2 | 3 | 4 | 6 | | Estimated weight | 7 | 1 | 4 | 2 | 6 | 3 | 5 | $\sum d^2 = 12$ $r_s = 1 - \frac{6 \times 12}{7 \times 48}$ $= 0.785(7142857...)$ **awrt** 0.786 OR $\frac{11}{14}$	B1 B1 B1 M1 A1	Correct values for first row. Correct values for second row. Accept reverse ranks.
(b)	H_0: There is no association between the actual weight and estimated weights of the cows. H_1: There is a positive association between the actual weight and estimated weights of the cows. 5% 1-tail critical value = 0.6786 Reject H_0 This suggests there is a positive association between the actual and estimated weights.	B1 B1 E1	Do not allow correlation. Either "Reject H_0" or "Positive association" FT their r_s
(c)	It only shows they were good at putting the cows in weight order. The contestant may have been a long way out with their guesses.	E1	B1 Anything which implies that this only shows they can order the cows.
		[9]	

Qu. No.	Solution	Mark	Notes
5(a)	H_0: The data can be modelled by the Binomial distribution $B(6,0.6)$ H_1: The data cannot be modelled by the Binomial distribution $B(6,0.6)$	B1	Both
(b)(i)	Expected frequencies are $(d = (P(X = 3) \times 50)$ $d = 13.824$	B1	
	$(e = (P(X = 4) \times 50)$ $e = 15.552$	B1	
(ii)	Combine classes with expected frequencies less than 5	M1	SC for solution that does not combine classes or only combines some. (M0M1m1A0B1B1B1B1)
	Use of χ^2 stat = $\sum \frac{(O-E)^2}{E}$ or $\sum \frac{O^2}{E} - N$	M1	$= \frac{13^2}{8.96} + \frac{12^2}{13.824} + \frac{15^2}{15.552} + \frac{10^2}{11.664} - 50$
	$= \frac{(13-8.96)^2}{8.96} + \frac{(12-13.824)^2}{13.824} + \frac{(15-15.552)^2}{15.552} + \frac{(10-11.664)^2}{11.664}$	m1	
	$= 2.319(254605)$	A1	cao
	DF = 3	B1	*Example of SC FT their table. May see DF=6*
	10% CV = 6.251	B1	*FT their DF CV = 10.645*
	Since $2.319 < 6.251$ do not reject H_0.	B1	*Since 17.394 > 10.645 Reject H_0*
	Insufficient evidence to reject the binomial model B(6, 0.6)	B1	*There is sufficient evidence to reject the binomial model B(6,0.6)* Only award final B1 if previous 3 B1 awarded.
(c)	6 is the number of clients she sees in **one day** AND 0.6 is the probability of selling a policy to each client.	E1	Must state one day.
		[12]	

Number of policies sold	0,1 or 2	3	4	5 or 6
Observed	13	12	15	10
Expected	8.96	13.824	15.552	11.664

Qu. No.	Solution	Mark	Notes
6(a)	H_0: There is no association between highest level of education and salary. H_1: There is an association between highest level of education and salary.	B1	OR H_0: Highest level of education and salary are independent. H_1: Highest level of education and salary are not independent.
(b)	$k = \frac{108\times71}{664} = 11.54(8\ldots)$ or 11.55	M1 A1	Alternative method $71 - (49.4 + 10.05)$ OR $108 - (10.57 + 35.46 + 26.19 + 24.23)$
(c)	$m = \frac{(32-24.23)^2}{24.23}$ $m = 2.49166$ Accept $2.491659\ldots$ $n = \frac{(5-9.20)^2}{9.20}$ $n = 1.91739$ Accept $1.917391\ldots$	M1 A1	M1 either method correct. Both correct. NB Using more dp than in the expected values table gives **2.48798** and **1.91866**
(d)(i)	Add the chi squared contributions to get 19.61301	E1	
(ii)	Appropriate comment relating observed and expected values. Eg. Fewer than expected in the highest earning category. More than expected in the lowest earning category. Expected value does not deviate much from observed value for £20 000 - £60 000 but does for the other two.	E1	
(e)	Appropriate comment on p value. Eg. The p value is < 0.05 which implies there is an association between highest level of education attained and salary. e.g. At the 1% significance level there is no association between highest level of education attained and salary. e.g. Although it can be shown there is an association it does not imply that highest level of education attained leads to a higher paying job.	E1 **[8]**	May be given for $19.61 > 15.507$ implies there is an association between highest level of education attained and salary.

Qu. No.	Solution	Mark	Notes
7(a)	$b = \frac{49.4511}{3.48}$	M1	
	$b = 14.2(0977\ldots)$	A1	
	$a = \frac{898}{14} - 14.2(0977\ldots) \times \frac{46.2}{14}$	M1	
	$= 17.2(5061\ldots)$	A1	
	$y = 17.3 + 14.2x$	B1	B1 FT 'their' gradient and intercept.
(b)	The regression line x on y would be more appropriate. OR would not be able to rearrange this equation to find x from y.	E1	
	Gives a value outside the range. OR 90 is outside the range.	E1	
		[7]	